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Analysis of False-Semantic Proof Production in Undergraduate Mathematics Learning Based on APOS Theory

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Abstract: *Mathematical proof is a series of rules set based on the logic and deductive to show that the statement is true. In proof constructing, Weber offered two ways, namely : semantic proof production and syntactic proof production. Semantic proof is a construction of proof that uses examples, diagrams or other mathematical objects to help draw up a formal proof. In this study, we emphasize on using of incorrect example or diagram in preparation of proof, so-called false-semantic. Characteristics of student's thinking process in false-semantic proof construction are required to know the appropriate treatment can be determined. Research conducted on students of Mathematics Education at the state university in the province of Banten, Indonesia. The research data obtained by asking the students to solve mathematical proof task, and analyzing the data using APOS theory. The results showed that there is students who done a false-semantic proof production. The students have mistaken in giving an illustration related to the statement to be proved. Analyzing from the thought processes using APOS theory, there are two classifications thought processes of students in proof constructing using a false-semantic, namely: (1) students' thinking structure which imperfection in "Process", and (2) students' thinking structure which imperfection in "Action". These research results are used in providing input and designing in mathematical proof learning.*
Keywords - *False-semantic proof, Semantic proof production, APOS Theory, Proving*

I. Introduction

Proving is the center of activity in mathematics learning. As every student of mathematics, both in high school and in college, faced with the problem of understanding and compiling a mathematical proof based on their thinking stage. The ability of constructing proofs must be understood by the students of Mathematics Education to be able to understand mathematics in depth. In addition, the ability of a provision as a mathematics pre-service teacher in order to create learning situations that encourage students to reasoning and proof. This is in line with the recommendation of the NCTM [1] that stated "reasoning and proof are not special activities reserved for special times or special topics in the curriculum but should be a natural, ongoing part of classroom discussions, no matter what the topic is being studied".

In the students' thinking process in constructing proofs, Pinto and Tall [2] used the term 'natural' to describe the process of extracting meaning and the term 'formal' to the process of giving meaning that work formally , In addition to the term 'natural' and 'formal', Weber [3] added the idea of 'procedural' learning for students who are just trying to cope with formal definitions and proof with rote learning. Alcock and Weber [4] divided the student response to 'semantic' and 'syntactic', base on terms in language which basically refers to the meaning of language (semantic contents) and grammar (syntax). Alcock and Weber describes the syntactic approach as one strategy in mathematical proofs by working from a literal reading of the definitions involved and semantic approach as a strategy in utilizing the intuitive understanding of a concept. It is actually in line with the term extraction or extract-meaning and giving meaning in the category of 'formal' and 'natural'.

In this regard, according to Weber [5] stated that there are three strategies are usually done to prove. The first strategy is the strategy of procedural proof production. In a procedural strategy of proof production, the students proved by looking for proof of similar examples. Furthermore, the students modify it according to the statement to be proved. Topics that are usually done with this strategy is the limit of a sequence and limit-function. A second strategy is syntactic proof production. Syntactic proof production strategy is a strategy that began with the mathematical proofs collected some definitions and assumptions are appropriate to the problem, and then draw conclusions based on the definitions and assumptions by utilizing existing theorems and rules of logic. The third strategy is the semantic proof production. Semantic proof production strategy is the strategy of mathematical proofs that use different representations of mathematical concepts informally to guide the writing of strict and formal proof. The representation in the form of illustrations (graphs, drawings, or examples of cases). Through the help of illustrations, the idea of proving is expected to appear.

Based on our observation, founded that although students trying to make sense of the proof by way of illustration, but some of illustrations created these students are incorrect. This resulted in the next step of proof becomes problematic. Illustrations are incorrect as it is given name a "false-semantic". This article seeks to uncover the thought process in constructing proofs students who experience false-semantic.

In revealing the thinking process, this study is based on APOS theory [6]. APOS theory proposes that an individual has to have appropriate mental structures to make sense of a given mathematical concept. The mental structures refer to the likely actions, processes, objects and schema required to learn the concept. Research based on this theory requires that for a given concept the likely mental structures need to be detected, and then suitable learning activities should be designed to support the construction of these mental structures.

II. Method

2.1 Participants

Participants of this study were 6 students who selected from 71 students of 3rd year undergraduate students of developing university in Banten Province, Indonesia. They are selected because they have been used false-semantic on their proof.

2.2 Proving Task

The instrument used in this study is a task modified from Moore [7]. The question is " Suppose A, B, C ⊆ R, A,B,C non-empty set. Prove If B ⊆ C and A ∩ B ∩ C = A then A∪B∪C = C.

2.3 Data

The data in this article was obtained within 3 phases. In Phase 1 students were asked to prove of proving-task. This stage is expected to provoke students' thinking to construct mathematical proof. The proof produced by the students is a image of the students' thinking in constructing proofs. In Phase 2 is to select students who used false-semantic on their proof. And phase 3 is to analyze the students' proof using APOS theory.

2.4 APOS Indicator

Analysis of student's thinking process in constructing proof using APOS theory. In APOS theory, the main mental mechanisms for building the mental structures of action, process, object, and schema are called interiorisation and encapsulation [8]. The mental structures of action, process, object, and schema constitute the acronym APOS. APOS theory postulates that a mathematical concept develops as one tries to transform existing physical or mental objects. The descriptions of action, process, object and schema in this research are given below;

- Action: A transformation is first conceived as an action, when it is a reaction to stimuli which an individual perceives as external. Indicator of 'action' is student has given some correct-example for the proposition.
- Process: As an individual repeats and reflects on an action, it may be interiorised (coded : InterAct) into a mental process. A process is a mental structure that performs the same operation as the action, but wholly in the mind of the individual. Indicator of 'process' is student could modelled the task into mathematical equation using variables.
- Object: If one becomes aware of a process as a totality, realises that transformations can act on that totality and can actually construct such transformations (explicitly or in one's imagination), then we say the individual has encapsulated (coded : EncaPro) the process into a cognitive object. Indicator of 'object" is student could made representation which connected to other concepts.
- Schema: A mathematical topic often involves many actions, processes, and objects that need to be organised and linked into a coherent framework, called a schema. It is coherent in that it provides an individual with a way of deciding, when presented with a particular mathematical situation, whether the schema applies. For example, connecting between integer concept and divisibilty concept in the proposition .

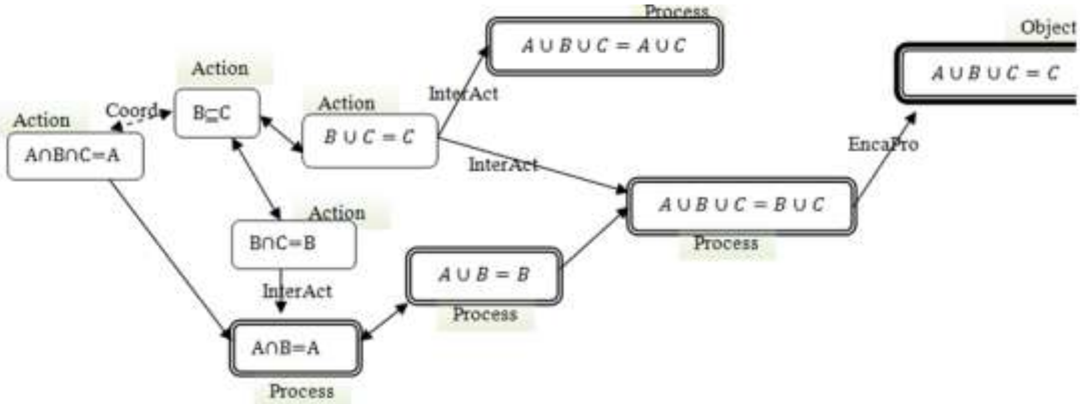


Fig. 1. Proof Structure for the proving task

III. Results And Discussion

Of the 71 students who were given the task of mathematical proof, there were 6 students who constructed proof through false-semantic, 33 students through true-semantic, 29 students through syntactic proof strategy, and 3 students who have no answer or blank. Based on the purpose of this article, it will be analyzed proofs from the six students who used false-semantic in constructing this mathematical proof.

The proofs constructed by the students have failed in constructing proofs. This is presumably because the student mistaken in giving an illustration related to the statement to be proved. In another sense, they used a false-semantic proof. Based on an analysis using APOS theory, there are two classifications thought processes of students in constructing proofs using a false-semantic proof, namely: (1) Students' thinking structure which imperfection in "Process", and (2) Students' thinking structure which imperfection in "Action".

1.1 Students' Thinking Structure which Imperfection in "Process"

There are two students who are thinking structure in this group, i.e. S1 and S2. In the proof-task, the hypothesis are $B \subseteq C$ and $A \cap B \cap C = A$. S1 stated that if $x \in A$ then $x \in A \cup B$. It is indicated that "Action" in thinking process is complete. With the result that interiorisation of 'Action' is completed. In the next step, S1 illustrated as follows :

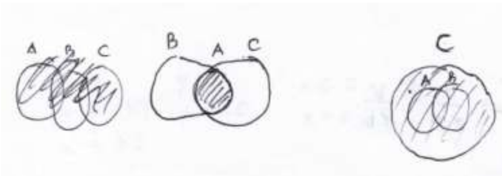
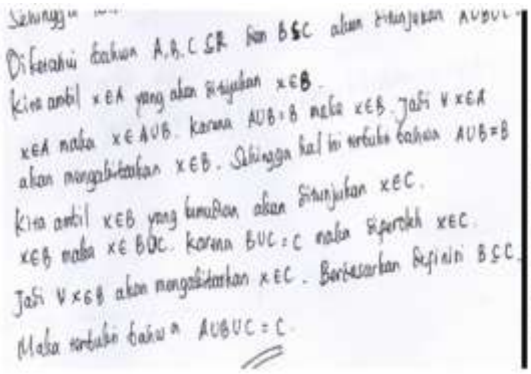


Fig. 2. Incorrect illustration which made by S1

The illustration show that $B \subseteq C$ and $A \subseteq C$, but $A \subseteq B$. With the result that S1 have fault in encapsulation this concept. Furthermore Encaplulation of "Process" to "Schema" becomes imperfect. The incorrect illustration caused fault in half step of proof.



Translate to English :
Suppose that A, B, C⊆R and B⊆C, will be shown that A∪B∪C=C. Let x∈A and we will show that x∈B. If x∈A then x∈A∪B. Because of A∪B=B, so x∈B. And so, if x∈A then x∈B. With the result that it's proved that A∪B=B. Let x∈B and we will show that x∈C. If x∈B then x∈B∪C. Because of B∪C=C, so x∈C. And so, if x∈B then x∈C. With the result that its proved that A∪B∪C=C.

Fig. 3. Proof Construction of S1

Proof construction of S2 has a fault, because it is not able to provide inferring well when $A \cup C = C$. The interiorisation of "Action" is good, because S2 could done $A \cup B \cup C = B \cup C$, whereas $B \subseteq C$. But, in encaplulation "Process" becomes "Schema" imperfect, because of incorrect illustration or false-semantic proof production.

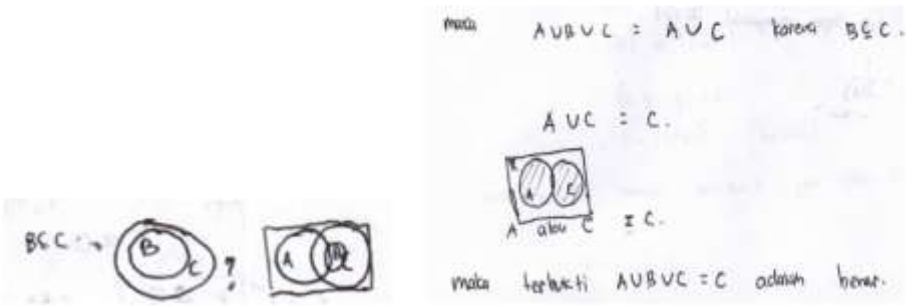


Fig. 4. Incorrect Illustration which made by S2 & Proof Construction of S2

Based on explanation above, we summarize the thinking structure when constructing proof using false semantic as follows :

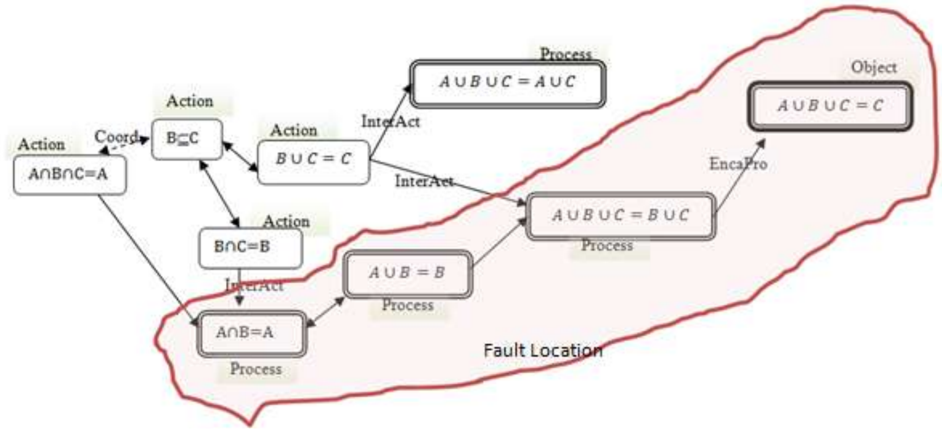
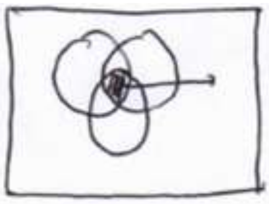


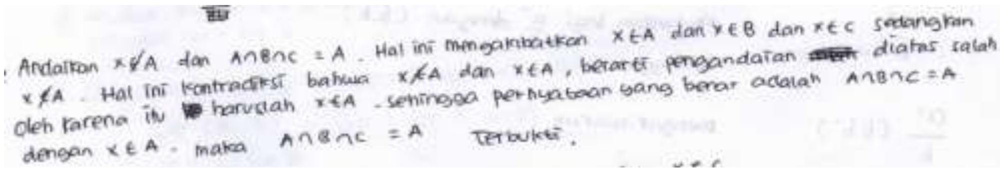
Fig. 5. False-semantic students' thinking structure which imperfection in "Process"

1.2 Students' Thinking Structure which Imperfection in "Action"

There are four students who structure their thinking in this group, namely : S3, S4, S5 and S6. Proof construction of S3 has a fault, because S3 wanted to prove by contradiction, but it is wrong in determining the premise is the basis for proving contradiction. The incorrect illustration caused fault in begin proof. If we use contradiction method in the task, we should taken hypothesis B⊆C and A∩B∩C=A and A⋃B⋃C≠C, but S3 used *x*∉A and A∩B∩C=A. And so, interiorisation from 'action' to 'process' is failed because of incorrect illustration.



(a)Incorrect illustration which made by S3



Translate to English : Supposed that $x \notin A$ *and* $A \cap B \cap C = A$. *This condition cause* $x \in A$ *and* $x \in B$ *and* $x \in C$. *Whereas* $x \notin A$. *And so it's contradiction that* $x \notin A$ *and* $x \in A$. *It means that the hypothesis is incorrect. Because of that, true proposition is* $A \cap B \cap C = A$ *with* $x \in A$, *and then* $A \cap B \cap C = A$. *Proved*

(b)Proof Construction of S3

Fig. 6. Proving Task Which Made By S3

Proof construction of S4 has a fault, because not able to extract the meaning A∩B∩C = A; B⊆C, so it does not happen interiorisation "Action" well.

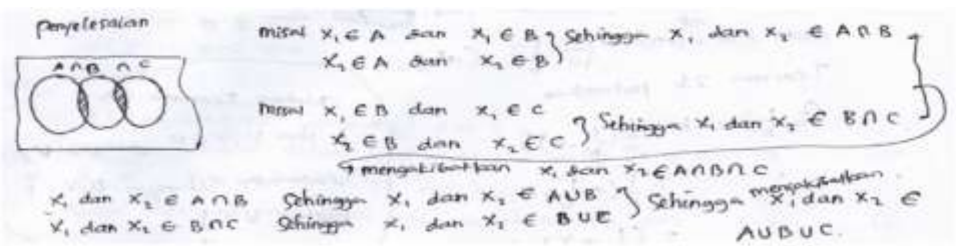


Fig. 7. Incorrect Illustration which made by S4 & Proof Construction of S4

Based on explanation above, we summarize the thinking structure when constructing proof using false semantic as follows :

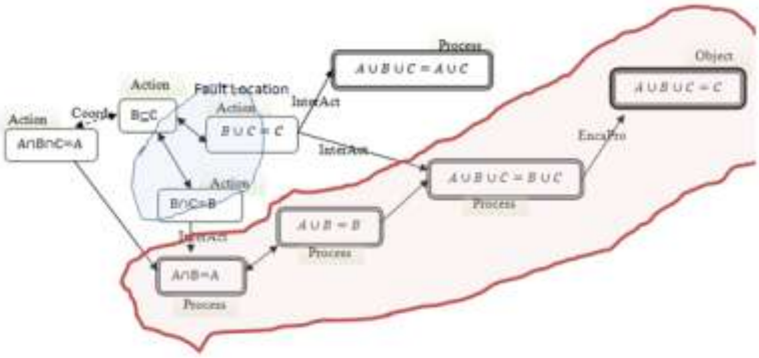


Fig. 8. False-semantic students' thinking structure which imperfection in "Action"

1.3 Learning Strategies to Students who Used False-semantic

According to Andrew [8], proof construction through false-semantic is poor in mathematical concepts and proof-structure. In addition, thinking structure have been made is uncompleted. Because of that, learning strategy must be suitable for refinement a mathematical concepts and proof-structure. Silver et al. [9] collected learning strategies and organized them into four distinct styles of instructionan. One of them is understanding style that seeks to expand students' capacities to reason and explain. This strategy seek to evoke and develop students' capacities to reason and use evidence and logic. Methods in understanding style are 'reading for meaning' and 'conceptual attainment'.

Reading for meaning is one method to encourage students about mathematical concept. Reading for meaning is a reading strategy that uses simple statements to help students find and evaluate evidence and build a thoughtful interpretation. The strategy engages students in the process known as "strategic reading." The strategy helps readers overcome common reading difficulties. Reading for Meaning statements are extraordinarily flexible tools for building students' reading skills.

Concept attainment strategy is also a good method for this condition. Because of this method is an in-depth approach to teaching and learning concepts based on the careful examination of examples and nonexamples. Concept Attainment is a strategy that allows students to explore critical concepts actively and deeply. The effectiveness of Concept Attainment as an instructional strategy is further bolstered by the fact that it engages students deeply in the skills of identifying similarities and differences and generating and testing hypotheses—two of the nine instructional techniques proven to raise students' level of achievement as identified by Marzano, Pickering, and Pollock [10].

IV. Conclusion

Based on the description above, it was concluded that there is a mathematical proof construction made by students through false-semantic. Proofs constructed by the students have failed in constructing proofs. This is presumably because the student mistaken in giving an illustration related to the statement to be proved. Analyzing from the thought processes using the theory APOS, there are two classifications thought processes of students in proof constructing using a false-semantic, namely: (1) students' thinking structure which imperfection in "Process", and (2) students' thinking structure which imperfection in "Action".

Learning strategy for this student condition must be suitable for refinement a mathematical concepts and proof-structure. The appropriate learning strategy is understanding style that seeks to expand students' capacities to reason and explain. This strategy seek to evoke and develop students' capacities to reason and use evidence and logic.

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